Confidence interval

A $1-\alpha$ confidence interval is an interval $[\hat{\Theta}^-, \hat{\Theta}^+]$ such that

$$ \mathbb{P}^\theta(\hat{\Theta}^- \leq \theta \leq \hat{\Theta}^+) \geq 1 - \alpha $$

for all θ.

In other words, there is a $1-\alpha$ probability that the generated confidence interval (random value based on sampling) captures the true value (deterministic).

Gaussian case

Assume:

$$ \hat{\Theta} \sim \mathcal{N}(\theta, \mathrm{se}^2) \sim \mathcal{N}(\theta, \hat{\mathrm{se}}^2) $$

Normal tables give us the following:

$$ \mathbb{P}\left(\left|\frac{\hat{\Theta} - \theta}{\hat{\mathrm{se}}} \right| \leq 1.96 \right) \approx 0.95 $$

$$ \mathbb{P}(\hat{\Theta} - 1.96 \hat{\mathrm{se}} \leq \theta \leq \hat{\Theta} + 1.96 \hat{\mathrm{se}}) \approx 0.95 $$

So the 95% confidence interval is:

$$ [\hat{\Theta} - 1.96 \hat{\mathrm{se}}, \hat{\Theta} + 1.96 \hat{\mathrm{se}}] $$

The approximation $\mathrm{se} \approx \hat{\mathrm{se}}$ may cause significant deviation.