

Confidence interval

A $1-\alpha$ confidence interval is an interval $[\hat{\theta}^-, \hat{\theta}^+]$ such that

$$\mathbb{P}^{\theta}(\hat{\theta}^- \leq \theta \leq \hat{\theta}^+) \geq 1 - \alpha$$

for all θ .

In other words, there is a $1-\alpha$ probability that the generated confidence interval (random value based on sampling) captures the true value (deterministic).

Gaussian case

Assume:

$$\hat{\theta} \sim \mathcal{N}(\theta, \sigma^2) \sim \mathcal{N}(\theta, \sigma^2)$$

Normal tables give us the following:

$$\mathbb{P}\left(\left|\frac{\hat{\theta} - \theta}{\sigma}\right| \leq 1.96\right) \approx 0.95$$

$$\mathbb{P}(\hat{\theta} - 1.96\sigma \leq \theta \leq \hat{\theta} + 1.96\sigma) \approx 0.95$$

So the 95% confidence interval is:

$$[\hat{\theta} - 1.96\sigma, \hat{\theta} + 1.96\sigma]$$

The approximation $\sigma \approx \hat{\sigma}$ may cause significant deviation.

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