Conversion between continuous-time and discrete-time signals

Continuous-to-discrete transformation

Consider a bandlimited continuous-time signal $x_c(t)$: that is, it has no frequency content for $|\omega| \geq \omega_c$.

This signal can be converted to a discrete-time (DT) signal $x_d[n]$ by sampling $x_c(t)$ at intervals of T seconds. This operation is known as continuous-to-discrete (C/D) transformation. In other words,

$$ x_d[n] = x_c(nT) $$

The following relations come from this sampling:

$$ n = \frac{t}{T} $$

- DT sample count is CT time divided by the sampling period.

$$ \Omega = \omega T $$

- DT frequency is CT frequency multiplied by the sampling period.

Nyquist's sampling theorem states: if the sampling frequency is greater than twice the highest frequency in a signal (ω_c in this example), then we can reconstruct the CT Fourier transform of a signal, and thus also the CT signal, from the DT Fourier transform of the sampled DT signal.

The **Nyquist rate** is twice the highest frequency in a signal. The sampling rate should be at least this value to avoid aliasing. This condition can also be written as:

$$ f_s = \frac{1}{T} \geq \frac{\omega_c}{\pi} = 2f_c $$

If this condition is met, then:

$$ X_d(e^{j\Omega}) = \frac{X_c(j\omega)}{T} $$

- The DT Fourier transform is equal to the CT Fourier transform divided by the sampling period.