Laplace transform

Laplace transforms turn time-domain functions, where t is the variable (time), into frequency-domain functions, where s is the variable (complex frequency).

Formal definition

$$F(s) = \int_{0}^{\infty} f(t)e^{-st} dt$$

Inverse Laplace transform

Similar to the Z-transform, we usually calculate the inverse Laplace transform by reorganizing the Laplace representation into a form we recognize with partial fractions and then pattern matching. Again, the time-domain representation depends on the desired region of convergence - the same Laplace domain representation can result in different time-domain representations, depending on the RoC.

Laplace domain representation	Region of convergence	Time-domain representation
$H(s)=\frac{1}{s-p}$	$s > p$	$h(t)=e^{pt}u(t)$
$H(s)=\frac{1}{s-p}$	$s < p$	$h(t)=-e^{pt}u(-t)$
$H(s) = 1$	All	$h(t) = \delta(t)$

Why Laplace transforms are cool

Integration in the time domain becomes division by s in the Laplace domain, and differentiation in the time domain becomes multiplication by s in the Laplace domain. This is useful for block diagrams.

References

- https://mathvault.ca/laplace-transform/