Region of convergence

RoC in the Z domain

The region of convergence (RoC) is defined as the set of z for which the z-transform (and by extension, its infinite series representation) of the signal converges/exists.

The ROC cannot include a pole, so any possible regions of convergence exclude the circles $|z| = |p|$ for all poles p. As a result, regions of convergence look like donuts (between two poles), circles (less than the smallest pole), or the entire z-plane excluding a circle (greater than the largest pole).

The actual region can be found by reducing the Z-transform formula to an infinite geometric series whose common ratio includes z. Recall that the absolute value of the common ratio must be less than one.

Different signals can have the same Z-transform. Two signals must have the same Z-transform and region of convergence to be identical.

Choosing RoC from given constraints (Z)

- A causal system, which is right-sided ($h[n]=0$ for $n < 0$) has a region of convergence that extends to infinity.
- A left-sided system has a region of convergence that includes $z=0$.
- A BIBO stable system has a region of convergence that includes the unit circle ($|z| = 1$).

RoC in the Laplace domain

The region of convergence in the Laplace domain is similar.

In the Laplace case, the possible regions of convergence exclude the lines $\operatorname{Re}(s) = \operatorname{Re}(p)$, for all poles p. These lines divide the complex s plane into possible regions of convergence.

Choosing RoC from given constraints (Laplace)

- A causal system has a region of convergence that extends to positive infinity.
- A BIBO stable system has a region of convergence that includes the imaginary axis $\operatorname{Re}(s) = 0$.