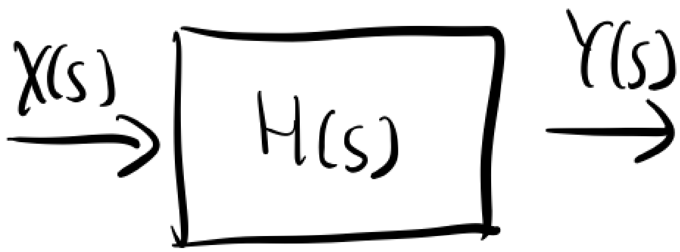
Transfer function

The transfer function of a system represents the ratio of its output to its input. Transfer functions are written in the Laplace transform or Z transform.



$$H(s) = \frac{Y(s)}{X(s)}$$

Examples of transfer functions

For a capacitor with capacitance C, let v, the voltage across the capacitor, be the input, and i, the current through the capacitor, be the output. Then,

$$H(s) = \frac{I(s)}{V(s)} = sC$$

since $\frac{I(s)}{V(s)}$ is the reciprocal of the impedance, $\frac{1}{sC}$.

For a mass m, let F, the force applied on the mass, be the input, and let x, the position of the mass, be the output.

$$H(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2}$$

since

$$F = ma = m\ddot{x}$$ $$F(s) = ms^2X(s)$$ $$\frac{X(s)}{F(s)} = \frac{1}{ms^2}$$

A bode_plot is a plot of the magnitude and phase of a transfer function as a function of frequency.

Zeros and poles

For a transfer function of the form

$$H(s) = \frac{N(s)}{D(s)}$$

Zeros are values of s for which $H(s) = 0$, and poles are values of s for which $H(s) = \infty$.

In general, an arbitrary transfer function with real coefficients can be reduced to a sum of first-order and second-order transfer functions. This is nice because finding the poles of first-order and second-order systems is easy.

If a system has a pole with a real part greater than zero (right half plane/RHP pole), then the system is unstable.

Standard form of the 2nd order system

Given a 2nd-order transfer function:

$$ G(s) = \frac{\omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2} $$

The poles are:

$$ p_{1,2} = \omega_n \left(-\zeta \pm j\sqrt{1 - \zeta^2} \right) $$

The system is:

- Overdamped if $\zeta > 1$
- Critically damped if $\zeta = 1$
- Underdamped if $\zeta < 1$

If the system is underdamped:

- The exponential decay constant σ is:

$$ \sigma = -\zeta \omega_n $$

- This exponential decay sets the envelope for the waveform.
- If the the system is underdamped, then the damped frequency is:

$$ \omega_n = \omega \sqrt{1-\zeta^2} $$