LTI filtering of WSS processes

Let $x(\dot)$ be a wide-sense stationary process with:

- Mean μ_x
- Autocorrelation $R_{xx}(\tau)$
- Autocovariance $C_{xx}(\tau)$
- $E[x^2(t)] \lt \infty$

Let $y(t) = h \ast x(t)$. Then, the following relations are true:

$$ E[y(t)] = H(j0) \mu_x $$

$$ R_{yx}(\tau) = h \ast R_{xx}(\tau) $$

$$ C_{yx}(\tau) = h \ast C_{xx} (\tau) $$

$$ R_{xy}(\tau) = \overleftarrow{h} \ast R_{xx}(\tau) $$

$$ C_{xy}(\tau) = \overleftarrow{h} \ast C_{xx}(\tau) $$

$$ R_{yy}(\tau) = h \ast \overleftarrow{h} \ast R_{xx}(\tau) $$

$$ R_{yy}(\tau) = h \ast \overleftarrow{h} \ast C_{xx}(\tau) $$

- $y(t)$ is also wide-sense stationary.
- $y(t)$ is jointly wide-sense stationary with its input.

General form:

Given $y = h \ast x$ and $z = g \ast w$:

$$ R_{yz}(\tau) = h \ast \overleftarrow{g} \ast R_{xw}(\tau) $$

Power spectral density

Main article: Power spectral density

CT case:

$$ R_{xx}(\tau) \leftrightarrow S_{xx}(j\omega) $$ $$ C_{xx}(\tau) \leftrightarrow D_{xx}(j\omega) $$

DT case:

$$ R_{xx}[m] \leftrightarrow S_{xx}(e^{j\Omega}) $$ $$ C_{xx}[m] \leftrightarrow D_{xx}(e^{j\Omega}) $$