Z-transform

The Z-transform converts a discrete-time signal into a complex frequency-domain representation.

It is the discrete-time equivalent of the Laplace transform.

Definition

Bilateral/two-sided Z-transform:

$$ x[n] \leftrightarrow X(z) $$

$$ X(z) = \mathcal{Z}\{x[n]\} = \sum_{n = -\infty}^{\infty} x[n]z^{-n} $$

$$ x[n] = \mathcal{Z}^{-1}\{x[n]\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(z) z^n d\Omega |_{z = \bar{r}e^{j\Omega}} $$

Inverse Z-transform

Usually, we will compute the inverse Z-transform by inspection, not by using the explicit formula.

For a rational Z-domain transfer function, this can be done by partial fractions. Separate the fraction into multiple terms, each of which corresponds to a single pole. Then, each of these terms can be transformed to the time domain. Keep in mind that the time domain function depends on the region of convergence.

Z-domain representation	Region of convergence	Time-domain representation
$H(z)=\frac{1}{z-p}$	$\|z\| > p$	$h[n]=p^{n-1}u[n-1]$
$H(z)=\frac{1}{z-p}$	$\|z\| < p$	$h[n]=-p^{n-1}u[-n]$